Non-Markovian quantum circuits can exhibit non-equilibrium steady states

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Why?

- Low-entropy, non-thermalizing quantum systems are not fully understood.
- Generating non-thermalized steady states → patterns in gate sequences. [1]
- How do non-Markovian (NM) processes influence the thermalization of a quantum system?

Non-Markovian sequence generation

- Markovian process: probability of each state in a sequence only depends on its previous state.
- n-gram model: probability of each state depends on the previous $n-1$ states.

given 3-gram rule

$$\begin{cases} (j,j) \to p = 0.30 \text{ for } j,\ 0.70 \text{ for } g \\ (j,g) \to p = 0.95 \text{ for } j,\ 0.05 \text{ for } g \\ (g,j) \to p = 0.05 \text{ for } j,\ 0.95 \text{ for } g \\ (g,g) \to p = 0.95 \text{ for } j,\ 0.05 \text{ for } g \end{cases}$$

possible generated sequences

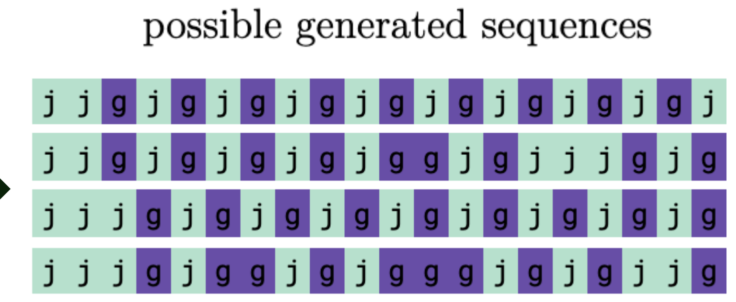

Figure 1. Example probabilistic 3-gram model for system with two states, 'j' and 'g'.

Qubit network simulation

- Constrained 8-qubit circuits dynamics + non-Markovian gate seqs. [1]

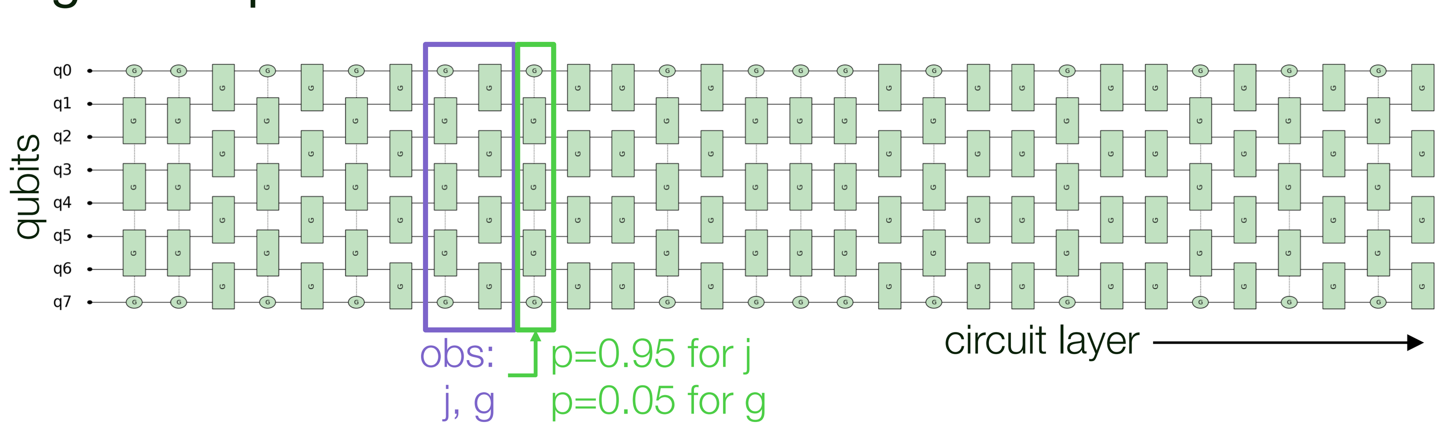

Figure 2. 3-gram non-Markovian gate sequences for connectivity C=2, 2-local 8-qubit network.

- Thermal resource initial state: each qubit is in mixed state.
  - One "hot" qubit with population $p_h = 0.4$.
  - Others are "cold" qubits with population $p_c = 0.1$.
- Gates evolve system with unitary of the form:

$$U_\star = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Rules simulated:

Baseline gate seqs	Non-Markovian gate seqs	
R1: random	R3: deterministic	R5: skewed probability
R2: brickwork	R4: even probability	R6: skewed probability

Extracting measures

- Any single-qubit state can be written in the Pauli basis

$$\rho_q(t) = \frac{1}{2}\left(\mathbb{1} + x_q\hat{\sigma}_x + y_q\hat{\sigma}_y + \boxed{z_q}\hat{\sigma}_z\right)$$

$$z_q = 1 - 2p_q$$

- During evolution, because of our constrained dynamics,

$$z_q(\ell+1) = \underbrace{\boxed{\tau_z}}_{\text{shift parameter}} + \lambda_z z_q(\ell)$$

- Correlation magnitude $\frac{1}{4}|C^{xx}_{ab}|$ between two qubits corresponds to coefficient of $(\sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y)$ term.

Quantifying out-of-equilibrium behavior

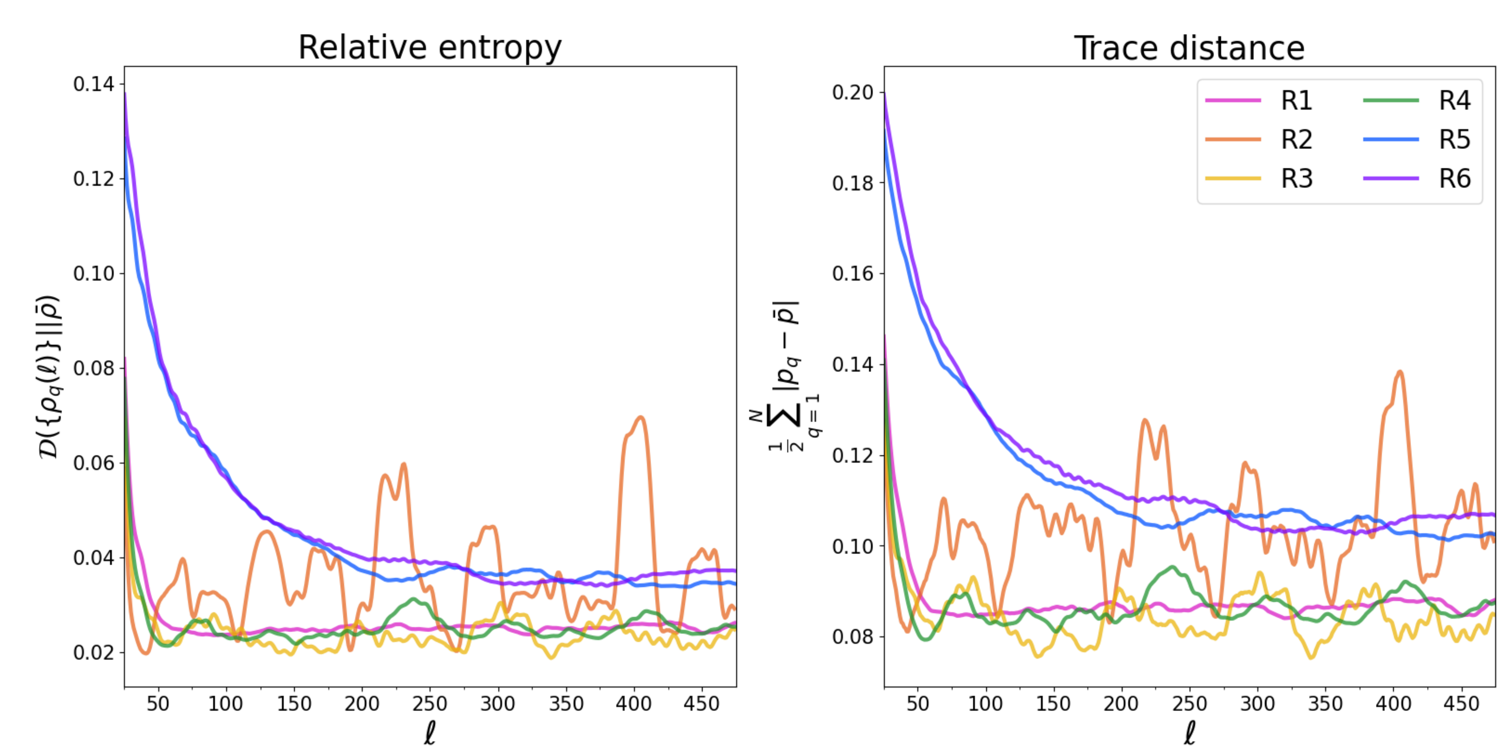

Figure 3. Ensemble-averaged relative entropy and trace distance as function of circuit layer, shown with moving average over 25 layers.

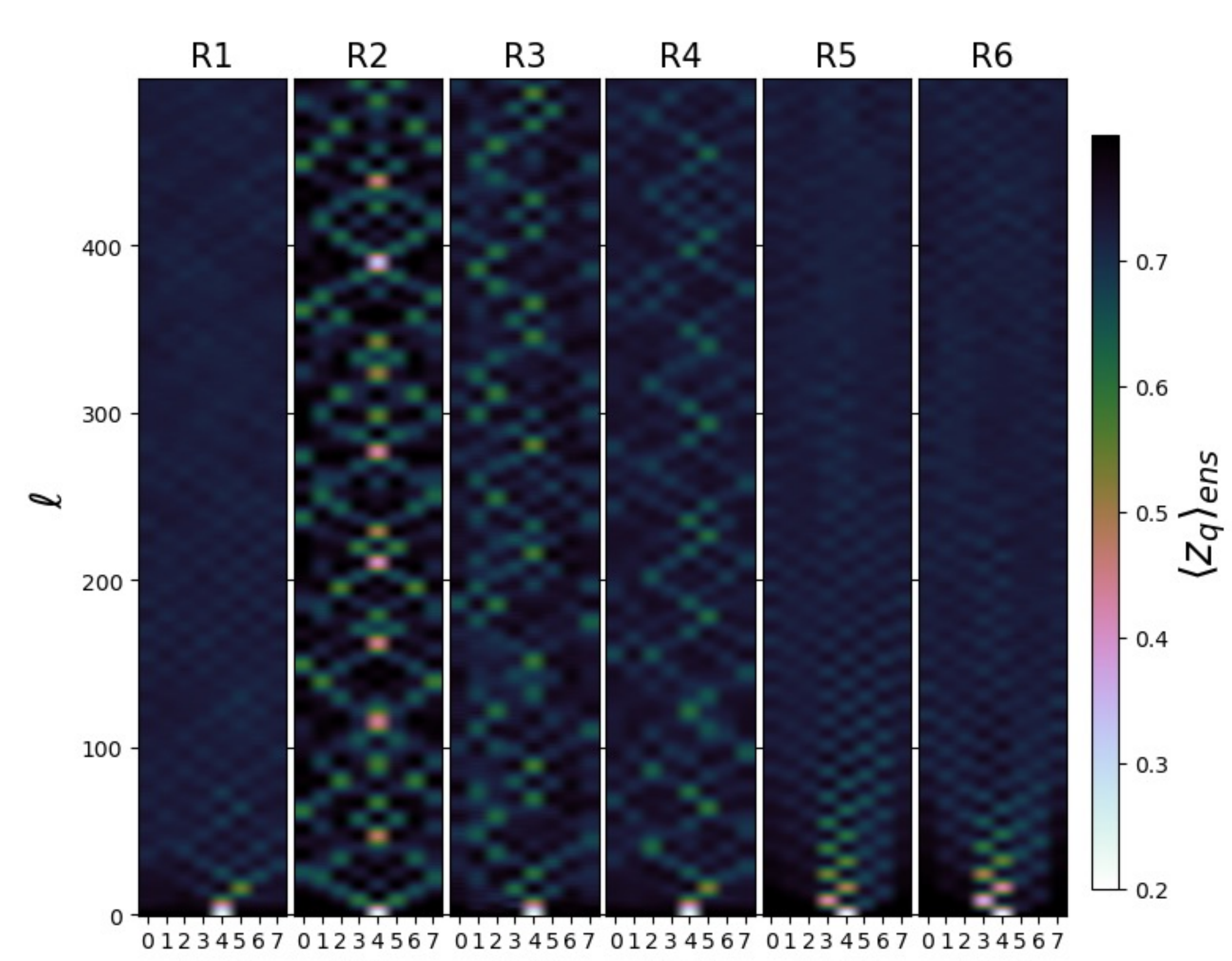

Figure 4. Ensemble-averaged $\langle z_q \rangle$ evolution heatmap for each rule (500 steps). Pixel color represents each qubit's value of z_q.

Connection to open quantum systems

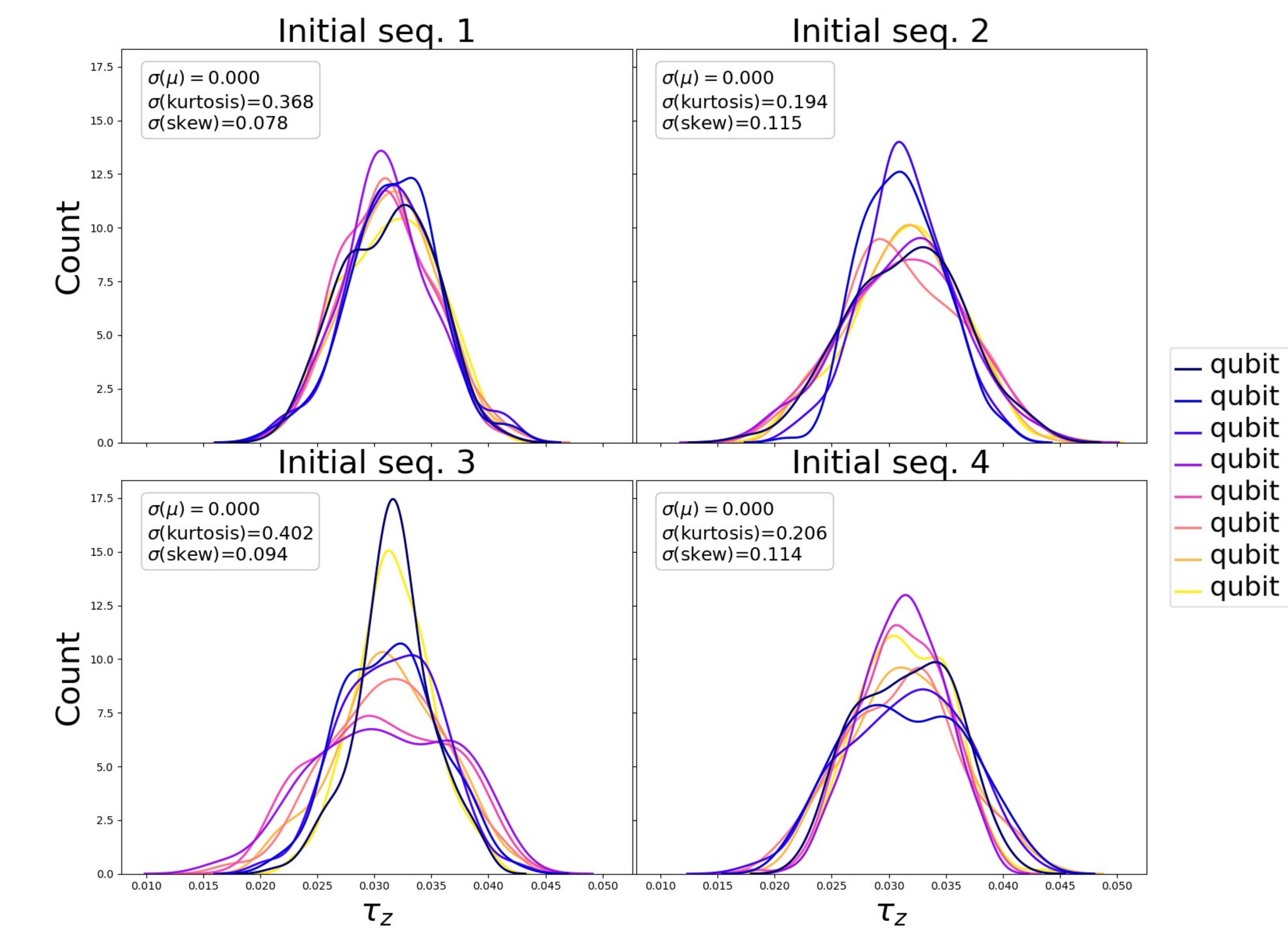

Figure 5. Ensemble-averaged τ_z distribution for R5, differentiated by initial gate sequences, with 25 trials for each initial sequence.

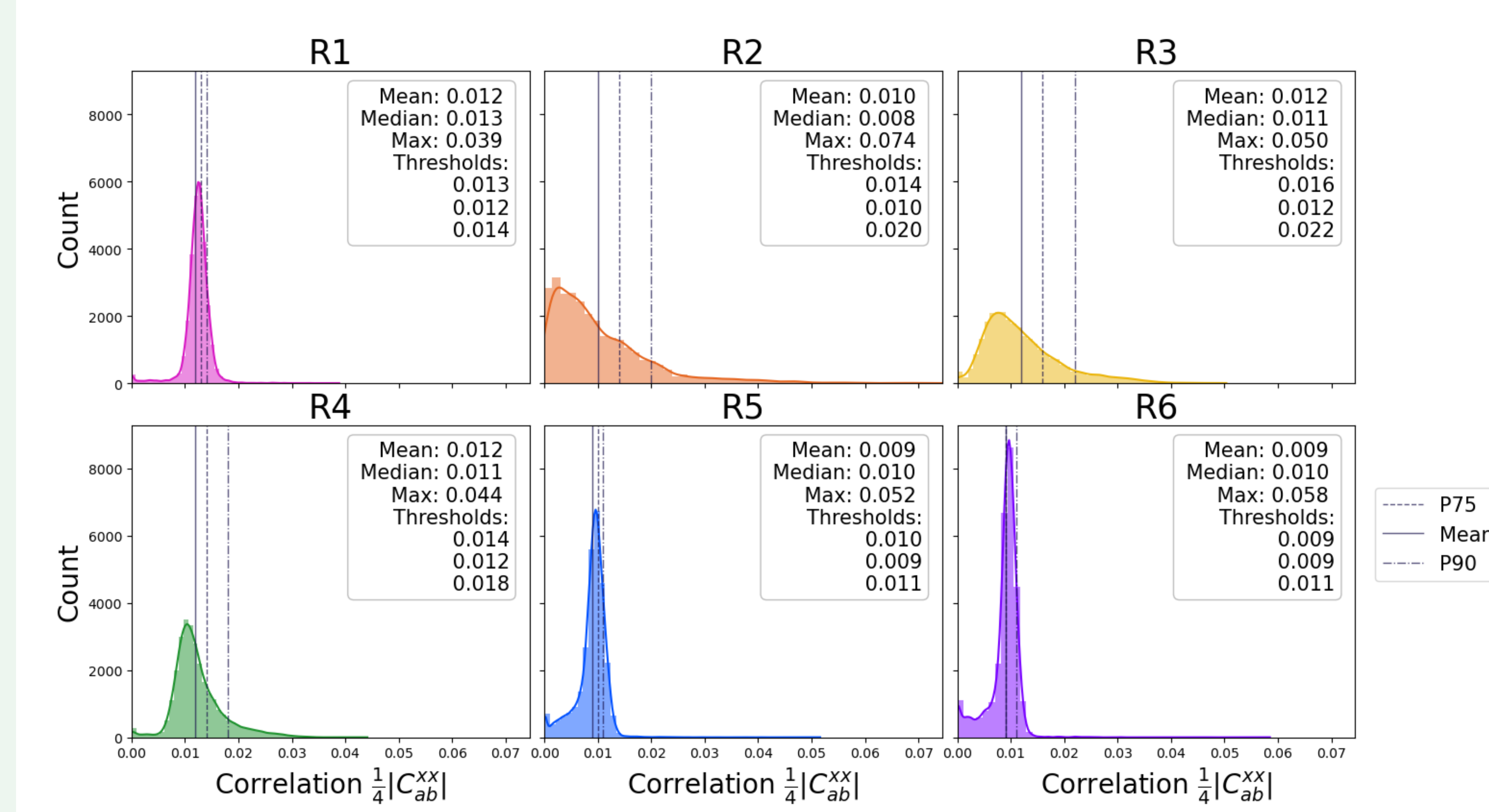

Figure 6. Correlation magnitudes distribution for evolution under each rule.

Conclusions

- Steady states are clearly non-thermalizing under some types of non-Markovian gate sequence evolution.
- New way of achieving non-thermalizing steady states, dependent only on circuit architecture.

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Reference

[1] Akhouri, U., Shandera, S. & Henry, J. Open-systems tools for non-thermalizing closed quantum systems. Preprint at arxiv.org/abs/2505.00116 (2025).

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